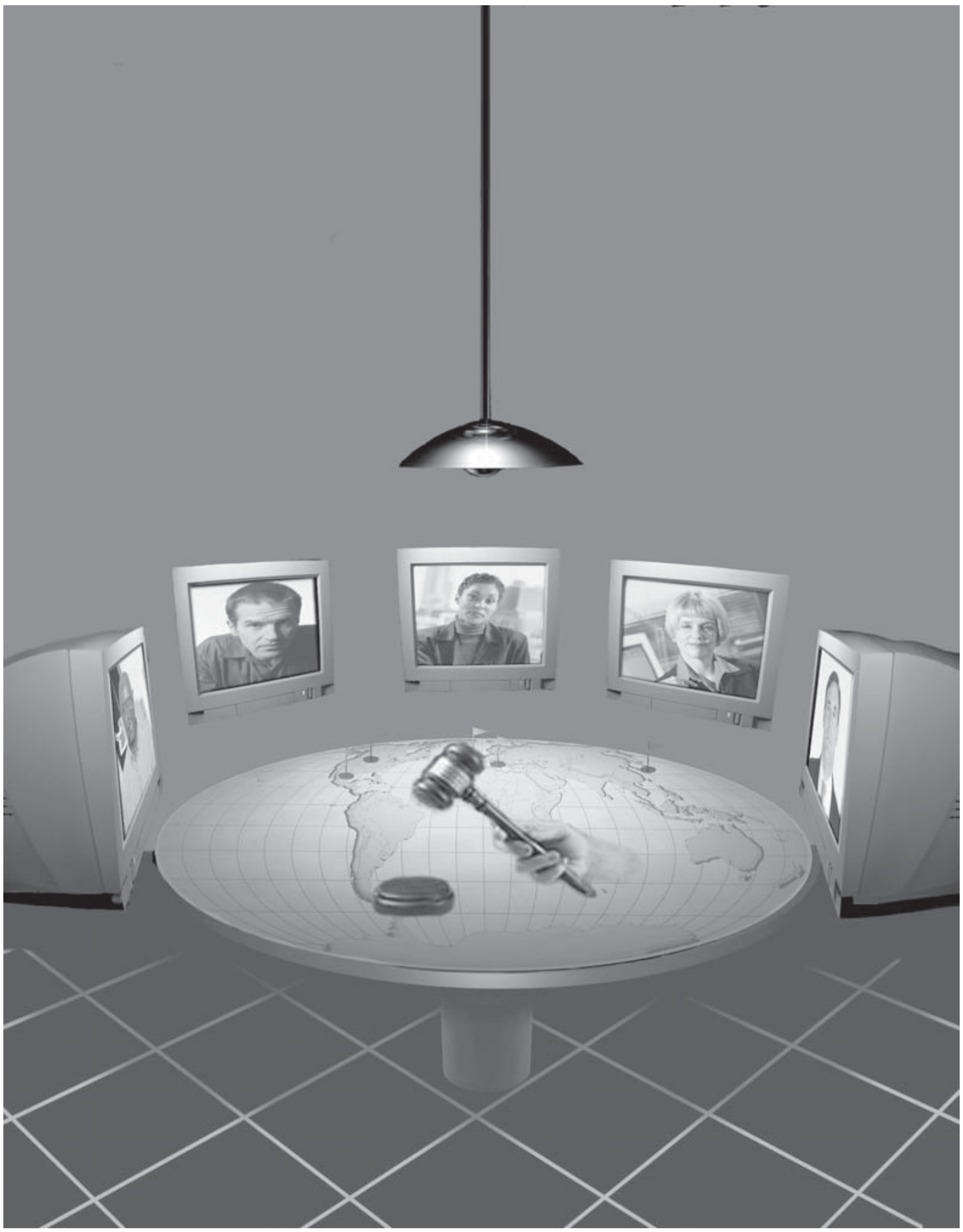

AUCTIONS IN DEFENSE ACQUISITION: *THEORY AND EXPERIMENTAL EVIDENCE*

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This article explores the implications of the theory and experimental evidence for auctions in the defense acquisition process. We begin with a brief review of the simplest auctions and proceed to recent theoretical/experimental results. The theoretical and experimental results discussed can shed light on efficient acquisition in the Department of Defense.

Auctions have been around for thousands of years. People buy and sell goods, services, and financial assets every day through auctions. More to the point, individuals and governments routinely use auctions to purchase goods and services from suppliers. Anyone wishing to see just what sorts of things are bought and sold by auction need only check one of the many online auction sites available today. When the Federal Acquisition Rules were rewritten in 1997 (Harris, 2001), the Office of Management and Budget eliminated the prohibition on auctions (Messmer, 2000). The enhanced technology currently available makes online auctions more appealing than ever, and the General Services Administration (GSA) has encouraged it's use.

The Department of Defense (DoD) has recently begun purchasing with online auctions — receiving offers from suppliers for things ranging from computers to equipment for U.S. Navy vessels. The Navy was the first to try online auctions for procuring airplane and ship parts. The Army's first venture into auctions was to purchase IBM ThinkPads, saving 40 percent off the standard GSA price. Since then, the Army has bought spare parts for the Patriot Missile system. The Air Force first tested the online auction waters in August 2000 to acquire computer equipment and saved about $88,000, or 27 percent of the estimated cost.

Now that auctions are possible, even encouraged, for DoD acquisition, the question arises as to how these auctions should be conducted. The purpose of this essay is to discuss the different types of auctions DoD may employ and describe the characteristics and qualities associated with each. When buying a single item (or lot of items), like a certain quantity of computers as in the Army case, what are the properties of a sealed-offer auction relative to a reverse auction conducted online? After discussing the theoretical properties of these auctions, we will review some of the pertinent experimental literature that may have something to say about each type of auction. Finally, we will conclude with a summary of the significance of these results for DoD auctions.

THE THEORY

The most commonly studied auctions are the sealed-bid auction and the English auction. These auctions, along with the complementary auctions designed to purchase items, are described in Table 1. It is not immediately clear that any of these auctions has an advantage over the others. The theory of sealed-bid and English auctions is well developed elsewhere.[1] In this section we discuss the types of auctions most likely used in government acquisition, sealed-offer and reverse auctions, and draw inferences for these auctions from the substantial literature for the complementary sealed-bid and English auctions.

Table 1. Description of Auction Types

Auction Type	Bidding/Offer Process	Description
English	bids increase	This is the typical auction in which a single seller of a single item (or lot of items) receives increasing bids from prospective buyers. The auction ends at a predetermined time, and the item goes to the highest bidder for the highest bid price.
Reverse	offers decrease	The exact opposite of the English auction. A single buyer of a single item (or lot of items) receives decreasing offers from prospective sellers. The auction ends at a predetermined time, and the item is purchased from lowest offerer for the lowest offer price.
Sealed-bid	Sealed bids	A single seller of a single item (or lot of items) receives sealed bids from potential buyers. Bids are unknown to all other bidders. The object goes to the high bidder for the highest bid price.
Sealed-offer	Sealed offers	A single buyer of a single item (or lot of items) receives sealed offers from potential sellers. Offers are unknown to all other offerers. The object goes to the high offerer for the lowest offer price.

A derivation of predicted behavior in sealed-offer and reverse auctions under the assumption of risk-neutrality is provided in Appendix A. It is important to note that in both types of auctions with risk-neutral suppliers the expected price of the item sold to the government is the same, although the bidding strategies are clearly different. The intuitive result that the expected offer price decreases as the number of suppliers increases is also made concrete. It is clear that the two types of auctions are, in this very simple environment, equivalent in terms of expected outcomes although they do not have the same optimal bidding/offer strategies. This result is well known in the auction literature as the revenue-equivalence result,[2] which in the context of the auctions we are discussing might better be called an expenditure-equivalence result. Note, however, that this expenditure-equivalence depends heavily on the risk-neutrality assumption. It can be shown that, whereas risk aversion will not alter the results for the reverse auction, the sealed-offer auction will generate lower expected offer prices when suppliers are risk averse.[3]

Although the sealed-price and English auctions have been the most commonly analyzed, the isomorphism between these auctions and their complements, the sealed-offer and reverse auctions, allows us to make inferences from the auction literature and apply it for acquisition purposes. Also, economists have conducted many experiments to test the theoretical predictions of behavior in auctions. In the next section we describe some of the experimental economics literature on auctions and how it relates to the kind of government auctions that interest us here.

THE EXPERIMENTAL EVIDENCE

In general, the experimental evidence does not address sealed-offer or reverse auctions directly. However, the fact that the experimental evidence suggests that bidding behavior in English auctions is consistent with theoretical predictions strongly indicates equilibrium behavior among suppliers in reverse auctions. The results of experimental sealed-bid auctions are consistent with risk-averse buyers, however. Specifically, Cox, Roberson, and Smith (1982) and Coppinger, Smith, and Titus (1980) reported higher than expected bids in their sealed-bid auction experiments — results consistent with risk-averse buyers. These results hold regardless of the number of bidders who participated. This leads us to expect risk-averse behavior among suppliers in the sealed-offer auctions and lower offer prices irrespective of the number of suppliers.

> **"It is clear that the two types of auctions are, in this very simple environment, equivalent in terms of expected outcomes...."**

We can measure efficiency in sealed-bid auctions by calculating the proportion of times that the bidder who values the object most wins the auction. Although the English auctions almost always end with the highest value bidder winning, an inefficient outcome took place in sealed-bid auctions about 12 percent of the time in the experiments conducted by Coppinger et al. (1980) as well as Cox et al. (1982). In terms of sealed-offer auctions, this means we can expect suppliers who do not have the lowest cost to offer the lowest price occasionally. Again, this result held irrespective of the number of participants.

To test the prediction that a larger number of participants leads to higher bids in the sealed-bid, Battalio, Kogut, and Meyer (1990) varied the number of bidders in a simple sealed-bid auction environment. The bidders were asked to submit bids for an item with the same value in two potential markets, one with 5 bidders and the other with 10 bidders. The actual market size was randomly determined after bids were made. Battalio et al. came as close as possible to holding all other things constant with this procedure. They reported that 86 percent of bidders made larger bids in the market size of 10 relative to the market of only 5. Sixty percent of the increases were statistically significant. The application to sealed-offer auctions is straightforward — an increase in the number of suppliers implies lower offer prices in sealed-offer auctions.

"Again, the experimental evidence supports the prediction that more participants in sealed-offer auctions will lead to lower offer prices."

In another study, Dyer, Kagel, and Levin (1989) asked subjects to tender bids contingent on the size of the market. Specifically, subjects in a sealed-bid auction experiment were asked to make a bid in a market of three bidders and one for a market of six. They found that bids were higher in the market of six than in the market of only three in about three-quarters of the cases. In only 3 percent of the cases were the bids different in the opposite direction. When they ignored the bidders with the lowest one-third of the valuations, the proportion that increased their bids jumped to 85 percent. Again, the experimental evidence supports the prediction that more participants in sealed-offer auctions will lead to lower offer prices.

McAfee and McMillan (1987b) and Matthews (1987) considered the effect of uncertainty with respect to the number of bidders in a market. They were able to show theoretically that with risk-neutral bidders in a sealed-bid auction the expected revenue (or expenditure in the sealed-offer auction) should not be affected by uncertainty about the number of bidders. Dyer et al. (1989) examined this result using their experimental results from the contingent bids. They found that uncertainty about the number of bidders in a market tends to increase the bids in these auctions and used this as evidence to support the claim that higher than predicted bidding in these auctions results at least partially from risk aversion. Again, the relevance to sealed-offer auctions is clear.

Summary and Conclusions

The theory of auctions has been developed and refined recently through the development of game theory and experimental economic methods. As the DoD shifts toward the use of auctions for purchasing goods and services, an understanding of these auctions becomes ever more important. This article examined some of the theory and experimental evidence with respect to auctions.

We first described the predicted bidding strategies in sealed-offer and reverse auctions. Although the expected results are the same for risk-neutral participants in both auctions, we pointed out that lower offer prices would obtain in the sealed-offer auctions with risk-averse suppliers. The experimental evidence suggests, however, that the two auctions will not yield the same offer prices, and suppliers in a sealed-offer auction will tend to offer lower prices more than predicted. This result is fully consistent with risk-averse behavior.

We also explored the importance of the number of bidders in an auction. Here the evidence from experimental auctions reflects the sort of behavior we predict from the theory. As the number of participants increases, the winning offer price decreases. When uncertainty over the number of participants is present, behavior consistent with risk-averse participants is once again apparent.

Although buying goods and services through auctions is relatively new to DoD, auctions have been studied in other contexts since game theory was introduced as an economic tool, and the use of laboratory experiments has become popular. Achieving efficiency or low price goals will be more likely in DoD auctions if we look to the economic literature for some insight.

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APPENDIX A

Suppose the government is buying a good through an auction in which potential suppliers submit a sealed offer for a price at which they will provide the good. We will assume initially that there are only two risk-neutral suppliers with different costs, which we assume to be independently and identically distributed according to a uniform distribution with support $[0,1]$. Strategies in this auction will be offer price functions that yield an offer price as a function of the supplier's actual cost.

It is not difficult to show that an equilibrium offer price function, or relationship between the supplier's cost and his offer price, in the auction described above is $p(c) = \frac{1+c}{2}$, where c is the cost to the supplier and $p(c)$ is the price paid by the government. To see this, consider a supplier's expected profit maximization problem. We will contemplate only affine (straight line) offer price functions; that is, we are thinking about offer price functions of the form $p(c) = \alpha + \beta c$ with $\alpha, \beta \geq 0$. Notice that this function is strictly increasing in cost. If suppliers employ this offer price function and costs are uniformly distributed on $[0,1]$, then the probability the supplier has a cost higher than c is just $1 - c$. Also, the increasing offer price function ensures that the seller with the lower cost will make the lower offer and win the auction. Therefore, the probability a seller with a cost c has the lower cost is $1 - c = 1 - \frac{p-\alpha}{\beta}$. The profit a supplier gets if he wins the auction is $\frac{1}{n+1}$. Hence, each supplier solves the following program and maximizes his expected profit: $\max_p \left(1 - \frac{p-\alpha}{\beta}\right)(p - c)$.

The first order maximization condition for a supplier, then, can be stated as $\beta + \alpha - 2p^* + c \equiv 0$. Since $p(c) = \alpha + \beta c$, it is easy to verify that when $\alpha = 1/2$ and $\beta = 1/2$ the first order conditions are satisfied. Hence, $p(c) = 1/2 + 1/2\, c = 1/2(1 + c)$ is an equilibrium offer price function under the conditions specified.

Now, suppose there are *n* suppliers of the item, and the costs are still identically and independently distributed from the uniform distribution on the interval $[0,1]$. The probability that a supplier wins the auction is now $(1-c)^{n-1} = \left[1 - \dfrac{p-\alpha}{\beta}\right]^{n-1}$. That is, the probability a supplier wins is once again just the probability it has the lowest cost. A supplier chooses *p* to maximize the following: $\left[1 - \dfrac{p-\alpha}{\beta}\right]^{n-1}(p-c)$. The first order condition is now $-(n-1)(\beta - p^* - \alpha)^{n-2}(p-c) + (\beta - p^* - \alpha)^{n-1} = 0$. Dividing through by $(\beta - p^* - \alpha)^{n-2}$ yields $\beta + \alpha + c(n-1) - np^* = 0$. Again, it is easy to verify that $\alpha = 1/n$ and $\beta = \dfrac{n-1}{n}$ satisfy the first order conditions because $p(c) = \alpha + \beta c$. That is, the equilibrium offer price function is: $p(c) = \dfrac{1}{n} + \dfrac{n-1}{n}c = \dfrac{1}{n}[1 - (n-1)c]$.

Before moving on to the reverse auction that is now being used by many government agencies, let us consider the expected results of these sealed-offer auctions. First, what is the expected price the government will pay? The buyer can expect to pay $\alpha + \beta c_l$ where c_l is the lowest cost. It can be shown that with *n* suppliers having costs drawn from a uniform distribution on the interval $[0,1]$, the expected value for the lowest cost will be $1 - \dfrac{n}{n+1} = \dfrac{1}{n+1}$. Hence, the expected price will be $\dfrac{1}{n} + \dfrac{n-1}{n(n+1)} = \dfrac{2}{n+1}$. Note that as the number of suppliers increases, the expected price goes to zero, the minimum possible cost. Also, the winning supplier's profit decreases similarly. The profit for the supplier with the winning offer price (lowest cost) is $\frac{2}{n+1} - \frac{1}{n+1} = \frac{1}{n+1}$. Here, as n gets large profits go to zero. Also, since each supplier has a $1/n$ probability of winning, a supplier's expected profit is $\frac{1}{n(n+1)}$.

Now we can compare these results to the reverse auction. Fortunately, the equilibrium strategy is much more easily determined in the reverse auction than in the sealed-offer auction. We make the same assumptions about the distribution of costs as above. Since suppliers see the price descending, the dominant strategy for a supplier is to continue to reduce the offer price by the minimum increment until the price falls below his cost. As in the sealed-offer auction, the supplier with the lowest cost wins the auction, but here he sells at a price equal to the next lowest cost.

In order to determine the expected price in a reverse auction, we begin by considering the case of two suppliers. From above, it is clear that the expected cost of the winning supplier is $\frac{1}{3}$. Conditional on the fact that the losing bidder did not win, the expected cost for the losing bidder is $\frac{2}{3}$. That is, in terms of expectation any cost between $\frac{1}{3}$ and 1 is equally likely, so the expected value will be the midpoint because costs are uniformly distributed. Hence, the expected price with two suppliers is $\frac{2}{3}$. Note that this is the same expected price as in the sealed-offer auction described above.

More generally, with n suppliers the expected cost for the winning supplier is $\frac{1}{n+1}$, and the expected cost for the next lowest cost supplier is

$$\left[1-\frac{n-1}{n}\right]\frac{n}{n+1}+\frac{1}{n+1}=\frac{1}{n+1}+\frac{1}{n+1}=\frac{2}{n+1}\ ,$$

which is exactly the same result we derived above. As before, the expected profit for the winning supplier is $\frac{2}{n+1}-\frac{1}{n+1}=\frac{1}{n+1}$.

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ENDNOTES

1. See, for example, McAfee & McMillan, 1987a.

2. See Davis & Holt, 1993, pp. 282–284.

3. See, for example, Davis & Holt, 1993, pp. 306.