A STATISTICAL APPROACH TO THE DEVELOPMENT OF PROGRESS PLANS UTILIZING BAYESIAN METHODS AND EXPERT JUDGMENT

Tiffany L. Lewis, Thomas Mazzuchi, and Shahram Sarkani

The development of progress plans for each identified technical performance parameter (TPP) is a critical element of technical performance measurement. The measured values of TPPs are referred to as technical performance measures (TPMs). These terms are used interchangeably; however, TPMs more directly reflect how technical progress and technical risk are measured and evaluated. Progress plans, or planned performance profiles, are crucial to effective risk assessment; however, methods for developing these plans are subjective in nature, have no statistical basis or criteria as a rule, and are not sufficiently addressed in literature. The methodology proposed herein for progress plan development will involve the elicitation of expert judgments to formulate probability distributions that reflect the expected values/estimates used to establish progress plans.

Keywords: Technical Performance Parameter (TPP), Technical Performance Measure (TPM), Planned Performance Profile, Bayesian Methods, Technical Risk
PROGRESS PLAN
DEVELOPMENT
The development of individual progress plans for each identified technical performance measure (TPM) is a critical element of technical performance measurement; however, the methods for developing these plans are subjective in nature, have no statistical basis or criteria as a rule, and are not sufficiently addressed in literature. This step is arguably considered the most critical aspect of technical performance measurement because it provides the basis for forecasting successful product development or failure; however, the absence of clearly defined processes to develop planned performance profiles is a void that exists and will be specifically addressed in this article. Bayesian Methods and the elicitation of expert judgments to formulate probability distributions that reflect the estimated values will be utilized to establish the performance profiles.

The Practical Software and Systems Measurement (PSM) process presented in Roedler and Jones (2005) is highly flexible and provides the foundation for the execution of Technical Performance Measurement. This process has been adopted by the International Council of Systems Engineering as an accepted practice, and tailoring this approach for a typical DoD program results in a five-phase process: identification, planning, measurement, review, and reporting. Each phase consists of multiple process steps, and the methodology discussed herein occurs during planning. Thus, the latter three phases are not addressed or discussed in this study.

Within the identification phase, the customer establishes technical goals and requirements, program priorities are defined, and TPMs are identified to support program goals and priorities ideally traceable to the lowest level work breakdown structure (WBS) work package elements.

The planning phase begins upon completion of the identification phase. During the planning phase, program goals are allocated by WBS work package with respect to budget, schedule, and the expected maturity of the technology under review. Additionally, during the planning phase, planned performance profiles and tolerance bands are developed for each TPM identified, initial risk assessments are conducted, and a technical performance baseline is developed.

Why a Statistical Approach is Needed

Establishing the technical baseline and individual progress plans for each identified TPM is a critical element of technical performance measurement and is the means by which technical progress and technical risk are measured and evaluated. This occurs during the planning phase as well as risk assessment, and these activities are consistent with proposed TPM implementation methodologies found in current literature such as Roedler and Jones (2005).
However, it is important to note that a review of current literature suggests that the processes for the implementation and execution of technical performance measurement on acquisition programs are not well defined. Coleman, Kulick, and Pisano (1996, p. 6) refer to them as being ad hoc to a significant degree due to the lack of formally established practices and processes in industry as well as in the DoD. The PSM guidebook entitled *Practical Software and Systems Measurement: A Foundation for Objective Project Management* (Bailey et al., 2000, pt. 4, p. 4-18) refers to the process for developing baseline plans as “Estimation” and states that poor estimates often lead to failed projects and result from the lack of systematic estimation processes as well as other contributing factors. Plans are developed largely through consensus by program management, developers, systems engineers, and subject matter experts who rely on knowledge from previous experience, historical data if available, and known cost and/or schedule baselines. These progress plans are crucial to effective risk assessment; however, the methods for developing them are subjective in nature, have no statistical basis or criteria as a rule, and are not sufficiently addressed in literature. This lack of formal processes to establish performance profiles predisposes them to an inherent degree of error and uncertainty, above and beyond that inherent, due to immature technical development, which translates into higher risk. Thus, employing a formal methodology utilizing an established statistical approach will minimize the level of uncertainty, thereby reducing risk exposure.

**Bayesian Framework**

The methodology proposed in this article utilizes an expert judgment model within a Bayesian framework for the more complex case of continuous probability distributions. The most general form of Bayes’ Theorem applies to discrete probability distributions, and relates the conditional and prior probabilities of two events using the following equation.

\[
P(A|B) = \frac{P(B|A) P(A)}{P(B)}
\]  

(1)

\(P(A|B)\) is considered to be the conditional probability of \(A\) given \(B\); \(P(B|A)\) is considered to be the likelihood of \(B\) given \(A\), \(P(A)\) is the prior probability of \(A\) (i.e., no information regarding \(B\) is considered), and \(P(B)\) is the prior or marginal probability of \(B\).

The expert judgment model utilized in this study for the case of continuous distributions was originally presented by Mosleh and Apostolakis (1986) to estimate seismic fragility curves (i.e., the con-
ditional probability that a seismic stress such as wind or earthquake will cause equipment to fail)—an application that, upon evaluation, proved to have similarities regarding the use of expert opinion elicitation for planned performance profile development. In both cases, the general Bayesian framework for continuous distributions uses the opinions of experts as “evidence,” and this evidence is used as input to the decision maker’s state of knowledge using Bayes’ Theorem. Bayes’ Theorem, in its general form for continuous probability distributions, follows:

\[ \text{Pr}(x \mid E) = k E^{-1} L(E \mid x) \text{Pr}_o(x) \] (2)

\( \text{Pr}_o(x) \) is the prior state of knowledge (prior distribution) regarding the unknown quantity \( x \) prior to obtaining opinions from the experts; \( E \) is the set of opinions provided by experts about the value of \( x \); \( L(E \mid x) \) = the likelihood the evidence \( E \) is true, given the true value of the unknown quantity is \( x \); \( \text{Pr}(x \mid E) \) is the decision maker’s posterior state of knowledge (posterior distribution) about the unknown quantity \( x \) given the set of opinions \( E \) provided by experts and a normalization factor \( k \), which is used to make \( \text{Pr}(x \mid E) \) a probability distribution. Within this framework, the likelihood function can be equated to the accuracy level of the expert’s estimate. Bayes’ Theorem can also be written as Equation 3 below where \( \alpha \) consists of the set of \( m \) parameters of the cumulative (unknown) distribution \( \Phi(x \mid \alpha) \).

\[ \text{Pr}(\alpha \mid E) = k^{-1} L(E \mid \alpha) \text{Pr}_o(\alpha) \] (3)

One unique curve \( \Phi(x \mid \alpha) \) is specified by each vector \( \alpha \). Thus, the average of the infinite number of distributions \( \Phi(x \mid \alpha) \) is defined by Equation 4 below.

\[ \Phi(x) \equiv \int_{\alpha} \Phi(x \mid \alpha) \text{Pr}(\alpha \mid E) d\alpha \] (4)

**Assumptions**

The following assumptions apply to the application of the Mosleh and Apostolakis (1986) model to the estimation of seismic fragility curves and also apply to estimation of progress plan values depicted in this illustration.

**No. 1.**

*The unknown distribution being estimated belongs to a parametric family of distributions.* This assumption simplifies the construction of the likelihood functions. As a result, the challenge
of estimating the unknown distribution is reduced to the estimation of its parameters.

No. 2.  
The unknown (posterior) distribution is assumed to be lognormal with parameters \( \Theta \) and \( \omega \). These two parameters are allowed to vary and describe the variability of the distribution. By definition, the minimum possible value for a lognormal distribution is zero, the maximum possible value is \( +\infty \), and the parameter values (mean and standard deviation) must be greater than zero. These criteria also apply to the TPM estimates used to establish a planned performance profile. Therefore, the lognormal distribution was deemed the most applicable distribution to use given the information available.

No. 3.  
Experts are independent and will be providing independently assessed percentiles. This assumption is intended to simplify the complexity of the model’s application to this problem. A model for the case of dependent experts is presented by Mosleh and Apostolakis (1986); however, it will not be explored here.

No. 4.  
Standard deviations assigned to each expert reflect the decision maker’s level of confidence in the expert’s ability to accurately estimate each percentile. If available, historical data reflecting the planned versus actual values of prior predictions should be used to determine the “bias” or “percent error” to associate with each expert’s accuracy and standard deviation.

No. 5.  
The percentiles being estimated are assumed to be symmetric. Symmetric percentiles (10 percent, 50 percent, and 90 percent) have been assumed for the model presented by Mosleh and Apostolakis (1986). Values for these percentiles are estimated by each expert.

No. 6.  
The standard deviations of the percentiles estimated by each expert are independent of the percentiles themselves. In other words, each expert is assumed to have the same level of accuracy estimating each percentile regardless of the percentile itself. Therefore, the standard deviations assigned to each expert for each percentile estimated would be the same. However, there is evidence to support that experts are more likely to be less accurate at higher percentiles, thus reflecting a greater level of uncertainty at these levels (George & Mensing, 1981).
Statistical Approach

The methodology proposed is presented to convey the feasibility of using an expert judgment model developed to estimate seismic fragility curves to develop retrospective progress plans. The development of progress plans is considered to be a key and critical process element of technical performance measurement.

The Cockpit-21 TPM project described in the unpublished white paper by Coleman et al. (1996) presents a suitable example and adequate data and information to convey how the Mosleh and Apostolakis (1986) methodology could have been employed to develop the planned performance profiles for each TPM identified. The example utilizes the model to formulate individual estimates for unknown distributions (as opposed to single values) based on estimates for multiple percentiles provided by experts. The estimated distributions reflect values of specific TPMs for different points in time that have been defined to coincide with key milestone dates. The following process steps are proposed for the development of progress plans and are based on the seismic fragility curve model.

**Step 1.** Identify TPMs.
**Step 2.** Define dates and milestones for the progress plans of each parameter.
**Step 3.** Identify experts that will participate in the estimation process.
**Step 4.** Assign weights to the experts identified.
**Step 5.** Assign standard deviations to the experts identified for each percentile estimated. Determine standard deviations for experts with unique weights.
**Step 6.** Experts estimate a TPM value for each established percentile (10 percent, 50 percent, and 90 percent) for each milestone date.
**Step 7.** Evaluate the standard deviation ($\sigma$) of the lognormal parameters ($\Theta$) and ($\omega$).
**Step 8.** Evaluate the value of the lognormal parameters ($\Theta$) and ($\omega$).
**Step 9.** Evaluate the distribution curve for each estimate using Bayes’ Theorem.
**Step 10.** Define tolerance bands for established performance profiles.
**Step 11.** Plot the mean values for each curve against its respective milestone date to establish the progress plan.
Identify Technical Performance Measures

Once goals and requirements have been set, TPMs are identified to support the program goals and priorities. The TPMs identified are assumed to have some notable impact to program costs, critical path, or technical risk. Two technical areas representing 50 percent each of the technical performance baseline were identified for the Cockpit-21 project. These areas were the Display Electronics Unit (DEU) software and Flight Test Problem Reporting. Table 1 displays the technical parameters and sub-parameters identified for DEU and for Flight Test Problem Reporting. Additionally, the weight of each parameter at each level is shown.

Define Dates and Milestones for the Progress Plans

The TPM milestone assessment dates for the Cockpit-21 program ranged from June 1992 through January of 1995, and the date range for each parameter varied with respect to its scheduled develop-
ments within the project’s life cycle. Assessments were scheduled at monthly intervals for each parameter; however, Roedler and Jones (2005) indicate that it is advisable to utilize significant milestones and/or design events to establish performance profile measurement dates. Coleman et al. (1996) also states that dates should be based on events during the development cycle as opposed to a periodic scheme. Therefore, this would be the preferred method for establishing evaluation dates for TPM values when implementing technical performance measurement on future projects. The range of dates and number of estimates associated with each progress plan developed for the Cockpit-21 program are depicted in Table 2. Since TPMs are tracked at the lowest level, it is only necessary to develop performance profiles for the lowest level parameter. Thus, for the purpose of this application, progress plans are not developed for level 2 parameters that have level 3 subparameters; those level 2 parameters are not depicted in Table 2.

### Identify Experts

The experts involved in the TPM planning process on a DoD program will most often be the members of the integrated product teams (IPTs) established to test, monitor, and evaluate technical progress. IPTs will primarily consist of contractors/developers. However, government representatives may work with contractors to share responsibilities pertaining to the planning process. Members of each IPT are selected based on their knowledge and experience.

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**TABLE 2. COCKPIT-21 PROGRESS PLAN EVALUATION DATES**

<table>
<thead>
<tr>
<th>Display Electronics Unit (DEU) Parameters</th>
<th>Start Date</th>
<th>Finish Date</th>
<th># of Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 HUD Design and Code Status</td>
<td>June ‘93</td>
<td>Feb ‘94</td>
<td>9</td>
</tr>
<tr>
<td>1.2 HUD Full Qualification Test (FQT) Status</td>
<td>June ‘93</td>
<td>Feb ‘94</td>
<td>9</td>
</tr>
<tr>
<td>2. Manpower</td>
<td>June ‘92</td>
<td>Feb ‘94</td>
<td>21</td>
</tr>
<tr>
<td>3.1 MFD Design and Code Status</td>
<td>June ‘93</td>
<td>Feb ‘94</td>
<td>9</td>
</tr>
<tr>
<td>3.2 MFD Full Qualification Test (FQT) Status</td>
<td>June ‘93</td>
<td>Feb ‘94</td>
<td>9</td>
</tr>
<tr>
<td>4. Requirements Volatility</td>
<td>Nov ‘92</td>
<td>Feb ‘94</td>
<td>16</td>
</tr>
<tr>
<td>5.1 Software Problem Reports Closed</td>
<td>Nov ‘92</td>
<td>Feb ‘94</td>
<td>16</td>
</tr>
<tr>
<td>5.2 Software Problem Reports Open</td>
<td>Oct ‘92</td>
<td>Feb ‘94</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flight Test Problem Reporting Parameters</th>
<th>Start Date</th>
<th>Finish Date</th>
<th># of Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>6. Reports Closed</td>
<td>Mar ‘94</td>
<td>Jan ‘95</td>
<td>11</td>
</tr>
<tr>
<td>7. Reports Open</td>
<td>Mar ‘94</td>
<td>Jan ‘95</td>
<td>11</td>
</tr>
</tbody>
</table>
regarding previous development efforts with the same or similar technology and may be systems engineers, test engineers, researchers, or analysts. For the purpose of this study, the IPT members are considered to be the experts, and the IPT leads are considered to be more knowledgeable than the other members of the team.

The question of how many experts are optimal was addressed by K. Walker (personal communication, November 11, 2004). She concluded there has been no evidence of an existing argument for an ideally statistically based sample size. Her literature review of 38 studies revealed 90 percent used 11 or less experts, and referenced Steve Hora, who has been repeatedly quoted for his argument of “three and seldom more than six” is sufficient (Hora, 2004, p. 5; K. Walker, personal communication, November 11, 2004). The example presented in this study assumes four experts are participating in the progress plan development process as members of an IPT.

Assign Weights to the Experts

Weights are assigned by the decision maker to each expert providing an estimate as a means to place more or less value on the responses of experts that are assumed to have greater or lesser accuracy with respect to their estimation ability. The more confidence a decision maker has in a particular expert’s ability, the more weight the expert’s estimate should carry with respect to other experts. The weights of all experts should be normalized so that they sum to one. If available, this step is where it would be appropriate to review historical data from previous projects to evaluate the estimation accuracy of experts conducting the estimates to determine the appropriate weighting scheme. Additionally, the experience level of each expert (or IPT member) and their level of knowledge regarding the technical area being evaluated should be considered when assigning weights. The assessment by the decision maker in this example assumes each expert is equally weighted with the exception of the IPT lead. The lead is given more weight to compensate for additional knowledge and experience in the technical area of interest.

Assign Standard Deviations to the Experts

Standard deviations are assigned by the decision maker to each percentile estimated by each expert. Each standard deviation reflects the perceived range of deviation from the true value being estimated at that percentile. The approach to assigning standard deviations is similar to that of assigning weights in that historical data from previous projects should be reviewed, if available, to
evaluate the true estimation accuracy or percentage error realized by experts with prior predictions. This should ensure the proper calibration of the experts and allow the decision maker to account for overconfidence of the experts as discussed by Hora (2004), and errors of estimation as discussed by Winkler (1981). In the case of the Cockpit-21 project, data from previous TPM pilot implementation projects discussed by Coleman et al. (1996), such as the Air Deployable Active Receiver (ADAR) sonobuoy development program and the LAMPS Block-II Upgrade program, would serve as appropriate reference materials for this effort. The model presented in this study assumes the standard deviations of all experts are equal with the exception of the IPT lead, who has a greater weight than the remaining team members (i.e., \( \sigma_{jk} = \sigma_j \)) where \( J \) equals the \( Jth \) percentile and \( (K) \) equals the \( Kth \) expert. To calculate the standard deviation of the IPT lead, Equation 5a must be used where \( N \) = the total number of experts, \( \sigma_k \) = the standard deviation of expert \( (K) \), and \( w \) = the weight of expert \( (K) \).

\[
N^{-1} \sum_{k=1}^{N} \sigma_k^{-2} = \frac{\sigma_j^{-2}}{w_j} \quad \text{(5a)}
\]

This will result in the standard deviation of the IPT lead being less than the remaining IPT members due to the lead being assigned a heavier weight.

**Estimate TPM Values by Percentile**

The methodology proposed here requires TPM values to be estimated by each expert for each established percentile (10 percent, 50 percent, and 90 percent) by milestone date. Meaning, each expert must predict the true value that has a 10 percent likelihood of being observed, a 50 percent likelihood of being observed, and a 90 percent likelihood of being observed for the given evaluation date. Experts formulate estimates based on the information they have available to them at the time, and this information may include historical and/or current test data, the results of formal functional analysis, and expert opinion based on knowledge and experience.

Table 3 depicts the proposed estimations for the first milestone date of the first technical parameter shown in Table 2 (Display 1.1, HUD Design and Code Status), the basis for the response provided, as well as the assigned weight and standard deviation for each expert. Equation 5a was used to estimate \( \sigma_j \) as follows:
TABLE 3. HUD DESIGN AND CODE STATUS EXPERT OPINION DATA

<table>
<thead>
<tr>
<th>Expert</th>
<th>wt.</th>
<th>dev. (σK)</th>
<th>Percentiles</th>
<th>Basis for Response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>10% 50% 90%</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>.24</td>
<td>0.100</td>
<td>22.0 23.0 26.0</td>
<td>test data, analytical methods</td>
</tr>
<tr>
<td>2</td>
<td>.24</td>
<td>0.100</td>
<td>21.5 22.8 24.0</td>
<td>test data, analytical methods, expert opinion</td>
</tr>
<tr>
<td>3</td>
<td>.24</td>
<td>0.100</td>
<td>22.0 23.5 24.5</td>
<td>test data, analytical methods</td>
</tr>
<tr>
<td>4</td>
<td>.28</td>
<td>0.093</td>
<td>21.5 24.0 25.0</td>
<td>test data, analytical methods, expert opinion</td>
</tr>
</tbody>
</table>

Using the values in Table 2, it is evident that each expert must provide 27 unique estimates (9 estimates x 3 percentiles) to establish a progress plan for this parameter; and a total of 12 estimates (3 percentiles x 4 experts) will be used in the expert judgment model to establish the unknown distribution for each individual estimate. To complete the expert judgment analysis for all parameters and all evaluation dates, this process would be completed for each parameter shown in Table 2, resulting in each expert providing 378 unique estimates (126 estimate dates x 3 percentiles per date).

Evaluate the Standard Deviation of the TPM Values

The unknown distribution associated with each estimate is assumed to be lognormal with parameters (σ) and (ω). The Mosleh and Apostolakis (1986) model uses Equations 6 and 7 to evaluate the values of the standard deviations for (Θ) and (ω). The standard deviation for (Θ) is denoted by Equation 6.
\[ \sigma_\theta^2 = \left[ 3 \sum_{K=1}^{N} \sigma_K^{-2} \right]^{-1} \]  

(6)

\[ \sigma_\omega^2 = \left[ 2Z_{0.99}^2 \sum_{K=1}^{N} \sigma_K^{-2} \right]^{-1} \]  

(7)

\( \sigma_\theta \) reflects the standard deviation for \( \theta \). The standard deviation for \( \omega \) is denoted by Equation 7, where \( \sigma_\omega \) reflects the standard deviation of \( \omega \) and \( Z_{0.99} = 1.285 \) from tables of the standard normal distribution.

\section*{Evaluate the Value of the Lognormal Parameters}

The unknown distribution associated with each estimate is assumed to be lognormal with parameters \( \theta \) and \( \omega \). The Mosleh and Apostolakis (1986) model uses Equations 8 and 9 to evaluate the values of the parameters \( \theta \) and \( \omega \). The parameter value for \( \theta \) is denoted by Equation 8.

\[ \Theta_m = \frac{1}{3} \sum_{K=1}^{N} w_K (\ln x_{70K} + \ln x_{50K} + \ln x_{90K}) \]  

(8)

The parameter value for \( \omega \) is denoted by Equation 9.

\[ \omega_m = \frac{1}{2Z_{0.99}} \sum_{K=1}^{N} w_K (\ln x_{90K} - \ln x_{10K}) \]  

(9)

\section*{Evaluate Each Unknown Distribution Using Bayes' Theorem}

Now that weights and standard deviations have been assigned to the experts, values have been estimated for each percentile by the experts, the standard deviations of the parameter values have been evaluated, and the parameter values themselves have been evaluated, all requisite information is available to evaluate the value of the unknown (posterior) distribution for the parameter depicted in Table 3. Using the Mosleh and Apostolakis model for Bayes’ Theorem, the posterior distribution can be evaluated using Equation 10.
\[ Pr(\Theta, \omega \mid E) = k^{-1} \exp \left\{ -\frac{1}{2} \left[ \left( \frac{\omega - \omega_m}{\sigma_{\omega}} \right)^2 + \left( \frac{\Theta - \Theta_m}{\sigma_{\Theta}} \right)^2 \right] \right\} \] (10)

Upon evaluation of Equation 10 for the parameter depicted in Table 3, and assuming the same experts are providing all estimates, steps 7 through 9 are repeated for the remaining parameters identified in Table 2 to complete the expert judgment analysis.

**Define Tolerance Bands for Performance Profiles**

Following completion of the expert judgment analysis, the next step is to define tolerance bands. Tolerance bands reflect the allowable range of variation and level of acceptable risk for a defined TPM estimate on a given milestone date. They alert management that actions may be necessary to get the TPM back on track. Actual values that exceed the allowable range (e.g., 20 percent) for any TPM estimate denote high risk (red) and will trigger management intervention. Additionally, allowable ranges are defined for low-risk items (green), and medium-risk items (yellow) as well. Typically, the bands depicted on a performance profile represent the maximum allowable variation.

**Plot the Expected Values for Each Curve**

If the expected values defined in the Cockpit-21 white paper are assumed to be the mean expected values determined using the expert judgment methodology, these values, along with the tolerance bands and the threshold values, would establish the performance baseline for each performance measure when plotted against their respective milestone dates. The Figure depicts the expected values used to define the individual progress plan for HUD Design and Code and the corresponding dates for each estimate. Actual values, a revised plan, and the threshold value are also plotted in the example shown for comparison with the baseline expected values.

**Discussion**

This article describes how expert judgment can be utilized within a Bayesian framework to develop a formal statistical model to quantify expert opinions as probability distributions for the purpose of establishing TPM progress plans. To demonstrate these attributes, key assumptions were made, and actual TPM data were
used to illustrate the application of the methodology. Baseline progress plans are crucial to effective risk assessment; however, the methods for developing these plans for each parameter are subjective in nature, have no statistical basis or criteria as a rule, and are not sufficiently addressed in literature. This lack of formal processes to establish performance profiles predisposes them to an inherent degree of error and uncertainty (i.e., risk). The formal methodology presented in this article offers an alternative that will arguably produce more accurate progress plans and will minimize the level of uncertainty, thereby resulting in reduced risk exposure. This proposed methodology provides program management with another decision-making tool that can be used to strengthen established systems engineering processes.
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\[
\frac{\theta(A/8)}{\theta(8)} = \frac{\theta(A|8)}{\theta(8)}
\]